Proceedings of Meetings on Acoustics

Volume 19, 2013 http://acousticalsociety.org/

ICA 2013 Montreal
Montreal, Canada
2 - 7 June 2013

Underwater Acoustics
Session 4pUW: Sparse Process Modeling Techniques for Acoustic Signal Processing

4pUW2. Application of statistical reduced isometry property to design of line arrays for compressive beamforming

Charles F. Gaumond* and Geoffrey Edelmann

***Corresponding author's address: Acoustics, U. S. Naval Research Laboratory , 4555 Overlook Ave SW Code 7165, Washington, D.C 20375, charles.gaumond@nrl.navy.mil**

The Statistical Reduced Isometry Property(StRIP) and Statistical Null Space Property (SNSP) are presented and reduced to numerical algorithms. These properties are used to predict the utility of a specific subsampled array for use in compressive sensing. Three examples of subsampling an equally spaced array are presented: random, Golomb and Wichmann. The Golomb array uses a Golomb ruler that has no repeated sensor element spacings. The Wichmann array includes at least one of every possible interval of sensor element spacings. The SNSP is shown to be insensitive to subsampling in the type of cases shown. The Golomb array is predicted to have superior performance to the Wichmann for comparable subsampling. The use of these two subsamplings for beamforming using at-sea data from the Five Octave Research Array (FORA) is shown. Research funded by the Office of Naval Research.

Published by the Acoustical Society of America through the American Institute of Physics

Received 22 Jan 2013; published 2 Jun 2013

INTRODUCTION

Early work in compressive sensing dealt with measurements that relied on the assumption of random sampling of the full aperture; in the case of beamforming, the array element positions to be analyzed are chosen randomly [Candes and Tao, 2005]. This assumption of randomness enabled mathematical proofs of perfect inversion of sparse signals. However, for practical reasons, array element positions are usually fixed, especially in the case of an underwater, acoustic array. More recent works contain mathematical proofs of sufficiency for perfect inversion with high certainty using assumptions applicable to deterministic arrays, namely arrays whose elements are fixed in place and could be placed through the use of a rule [Devore, 2007][Calderbank et al., 2010]. In both cases, specific array configurations based on coding theory are proven to enable perfect inversion with high certainty. The references present specific cases that are amenable to theoretical analysis. In the following sections, we present the numerical approach that allows the evaluation of more general cases; this numerical approach is generalizable to arbitrary array geometries and arbitrary statistical assumptions.

THEORY

Consider the acoustic problem where a subset of M hydrophones from N equally-spaced hydrophones is selected from a fully populated linear array at the design frequency of the array. This is described by the beamforming matrix $\Phi = e^{ikz_m n\Delta\cos\theta}$, where z_m is from a set of $M < N$ hydrophones chosen from $\{0,\Delta z,\ldots,(N\text{-}1)\Delta z\}$. The largest value of position and angular increment satisfy the criterion $N(\Delta z/\lambda)\Delta\cos\Theta=1$. This corresponds to the design frequency of an equally spaced array and to the finite Fourier transform. Beamforming with this rectangular matrix is defined as the inversion of

$$p(m) = \Phi(m,n)\,b(n), \tag{1}$$

where p is a vector of pressure measurements taken at M positions and b is a vector of $N>M$ beams. Compressive sensing techniques can be used if the beams b are assumed to be k-sparse [Edelmann and Gaumond, 2011]. The problem addressed here is the determination of the hydrophone positions $\{z_m\}$ that produce the most robust sparse inversions for a given number of hydrophones. The parameter commonly used to predict the invertibility of Φ is the Restricted Isometry Property (RIP) constant δ. Recently, the Statistical Restricted Isometry Property (StRIP) constant has been defined and applied to deterministic sets of sample points [Calderbank et al., 2010]. Using the same approach, we develop a numerical formulation to compare three types of arrays with fixed design.

The StRIP parameter is defined with the assumption of stochastic, k-sparse beam vector α^k and a deterministic, or fixed, set of spatial samples $\{z_m\}$. With a sufficiently large set of I^k random, k-sparse beam vectors $\{\alpha_i^k\}$, defined by a probability density function of source bearing $p_\alpha(\alpha^k)$, the set of corresponding values of the RIP parameter $\{\delta_i^k\}$ are found using the following usual definition of the RIP parameter

$$(1-\delta)\left\|\alpha_i^k\right\|_2^2 \le \left\|\Phi\alpha\right\|_2^2 \le \left(1+\delta_i^k\right)\left\|\alpha\right\|_2^2, \tag{2}$$

where Φ is assumed to be dimensionless. Following Dai and Milenkovic, the singular value decomposition is applied to Eq. 1, and $\left\|\Phi\alpha_i^k\right\| = \alpha_i^{k,T} V S^2 V^T \alpha_i^k$ [Dai and Milenkovic, 2009]. Then the ratio $(1\text{-}\delta_i^k)/(1+\delta_i^k)$ equals the condition C_i^k number of $V_i^k S^2 V_i^{k,T}$, where V_i^k contains only the non-zero rows in α_i^k. This leads to $\delta_i^k = (1\text{-}C_i^k)/(1+C_i^k)$. This formulation is non-dimensional. For the example of matched field processing in underwater acoustics, Φ is the propagation Green's function with units m^{-1}, α the source level with units $\mu Pa\text{-}m$. When Φ is rank deficient, which occurs when $k = M$, $\delta=1$. This set of values of the RIP parameter $\{\delta_i^k\}$ is used to estimate the cumulative distribution function of the random quantity, $p_\delta(\delta^k|\Phi,k)$, with an acceptable accuracy. With the assumption of a probability, for example π_δ=95%, a value of the StRIP parameter is determined from

$$P\left(\delta_\pi^k\right) = \pi_\delta. \tag{3}$$

Previous work has theoretically bounded the StRIP condition, but the bounds have been overly stringent requiring far lower values of sparseness than observed in practice. Here a numerical approach is developed that will yield bounds that are applicable to a wider variety of subsamplings. Note also that the above numerical approach can be used with a wide variety of assumptions, e.g., sound sources are more likely in specific directions such as a port.

NUMERICAL ANALYSIS

The following section will show the predicted invertability of several array geometries. Three statistical assumptions are made to accomplish this task. First, $p_\alpha(\alpha^k)$ is assumed to be equally distributed in cosine space (this is not required and if *a prioi* information is know *e.g.* sources are more likely towards a port that information can be used). Second, sufficient statistical convergence of $p_\delta(\delta^k|\Phi,k)$ is determined with the requirement that the greatest Kolmogorov-Smirnov difference between the i^{th} estimate and the previous ten estimates is less than a value of 0.001. Third, all interior array element positions are assumed to be equally probable for the selection of random array elements.

The random, Golomb and Wichmann subsamplings are each defined on an equally spaced array. The elements at the ends of the array are always included, to establish a fixed aperture, in each random array that has *M*-2 randomly placed interior elements. A Golomb ruler contains many unique intervals and contains no repeated intervals, whereas a Wichmann ruler contains all intervals but typically contains repetitions of some intervals. Each of these can be used for compressive sensing, but a comparison of StRIP is our basis for choosing which approach is better for given constraints of minimizing hydrophones or maximizing inversion convergence for a given aperture.

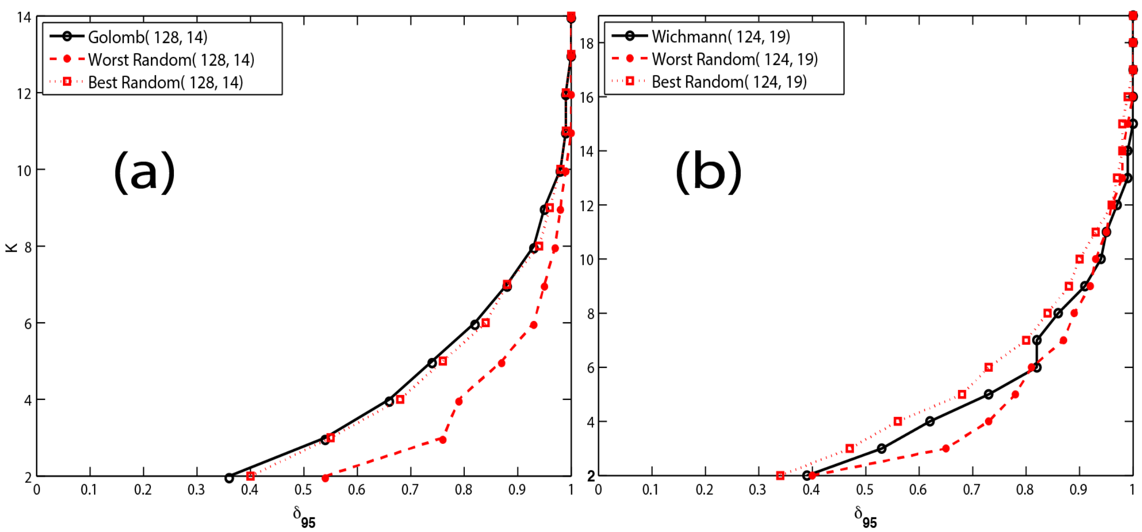

FIGURE 1. (a) The StRIP δ_{95} is plotted for the Golomb, best and worst of 200 random arrays. These arrays have length 128 with 14 elements. The Golomb array possesses a smaller StRIP than either of the two random arrays. However, the best random array has StRIP values very close to those of the Golomb array. (b) The StRIP δ_{95} is plotted for the Wichmann array, the best and worst of 200 random arrays. These arrays have lengths 124 with 19 elements. The Wichmann array has a StRIP that is never smaller than the best random array and is sometimes greater than the worst random array.

Not every length has an associated Golomb array. The particular Golomb array considered here has a non-dimensional length of 128 with 14 elements. It is then compared with two hundred random arrays of the same length and number of elements. The values of the StRIP parameters are shown in Fig. (1a). In this plot the vertical axis is the sparsity *k* of the solution and the horizontal axis is the StRIP parameter δ_{95}. The array with lower values of δ_{95} probably is likely to converge more quickly and more accurately. Note that the Golomb array has values similar to the best random array and considerably better values than the worst random array. The most recent, theoretically defined, sufficient bound for *k* is $k \leq 14/log(128) \approx 3$ for the Golomb array and $k \leq 19/log(128) \approx 4$ for the Wichmann [section IV, Calderbank, et al., 2010]. Careful numerical analysis predicts looser bounds of *k* for these two specific cases.

The Wichmann array is defined by the property that no integer inter-element interval is missing in the array although some may be repeated [Egidi and Manzini, 2011]. This arrangement can be found through the use of a formula. Like the Golomb array, a Wichmann array does not exist for every length. There is no Wichmann array with non-dimensional length 128. Therefore we consider the Wichmann array with slightly shorter length, 124, and with more elements, 19. The lowest and highest StRIP conditions of two hundred random arrays are also shown. The StRIP parameter δ_{95} of the Wichmann, best random and worst random is shown in Fig. (1b). Note that the Wichmann δ_{95} lies between the best and worst random arrays. Thus a random array may or may not converge better than the Wichmann array.

CONCLUSION

The new method presented here allows the numerical design of a specific compressive sensing array to be used for beamforming optimized for numerical invertability. Previous theoretical studies of the Fourier ensemble have dealt with random subsets of elements in a line array and have predicted requirements of more elements that are needed in practice. The StRIP condition is a well-defined, numerical method test that shows the comparative utilities of specific subsamplings of an array. The numerical approach is demonstrated for Golomb and Wichmann subsampled line arrays, and is generalizable to any geometric array configuration. Thus, this approach can be used to reduce the number of elements in combinations of linear arrays and also more complicated configurations, such as volumetric or vertical arrays in complicated environments.

ACKNOWLEDGMENTS

This work was funded by the Office of Naval Research.

REFERENCES

Candes, E.J., and Tao, T. (2005), “Decoding by linear programming,” IEEE Trans. Inf. Theory 51, pp. 4203-4215.

Calderbank, R., Howard, S., Jafarapour, S. (**2010**), “Construction of a large class of deterministic sensing matrices that satisfy a statistical isometry property,” IEEE J. Selected Topics in Sig. Proc. 4, pp. 358 – 374.

Dai, W., and Milenkovic, O. (**2009**), “Subspace pursuit for compressive sensing signal reconstruction,” IEEE Trans. Inf. Theory 55, pp. 2230-2249.

Devore, R.A. (**2007**), “Deterministic constructions of compressed sensing matrices,” J. Complexity 23, pp. 918-925.

Davenport, M.A., and Wakin, M.B. (**2010**), “Analysis of orthogonal matching pursuit using the restricted isometry property,” IEEE Trans. Inf. Theory 56, pp 4395-4401.

Edelmann, G.F., and Gaumond, C.F. (**2011**), “Beamforming using compressive sensing,” J. Acoust. Soc. Amer. 130, pp. EL232-EL237.

Egidi, L., and Manzini, G. (**2011**), “Spaced seeds design using perfect rulers,” Technical Report TR-INF-2011-06-01-UNIPMN, Computer Science Department, UPO, http://www.di.unipmn.it (Last viewed 18 Jan. 2013.)